(a) A cross-ply GFRP composite laminate (04/904)S is manufactured from material with moduli E1 = 40 GPa, E2 = 10 GPa, G12 = 4 GPa and ν12 = 0.3. Derive expressions for the Poisson’s ratio and the Young’s modulus of (04/904)S coupons machined from the laminate in terms of the reduced stiffness components Q11, Q12 and Q22 when loaded parallel to the 00 ply direction. Hence, find the numerical values of the Poisson’s ratio, νXY and the Young’s modulus, EX, for the laminate.

: Extensional stiffness matrix  
 : Coupling stiffness matrix : Bending Stiffness matrix.

For (04/904)S, = 0.  
If the laminate is loaded in-plane then = 0 and no curvature result. Hence, =

The basic stress/strain relations for a lamina loaded in its principal directions can be written as a reduced stiffness matrix:

The laminate considered is the following:

Now, if h is the thickness of a ply,

=

=

=

A stress σX is applied parallel to the 0° direction. Then, force/unit width = σX x 16h

Hence,

And,

σX x 16h =

0 =

As a consequence,

=

Finally,

σx x 16h =

Ex = =

Where:

= = 40.92 GPa

= = 10.23 GPa

= = 3.069 GPa

As a result,

Ex = 25.2 GPa

And νXY = - = = 0.12

(b) A mistake during the manufacture leads to the production of a (0/90)4S laminate instead of a (04/904)S laminate. Show that the Poisson’s ratio and the Young’s modulus of the (0/90)4S laminate are the same as for the (0/90)4S laminate (assume loading in the 0° direction in both cases). However, explain qualitatively why the flexural moduli of the two laminates will be different.

Equations are self explanatory:

Indeed, like it has been said at the beginning of this question, is the extensional stiffness matrix and is the bending stiffness matrix. The value of the flexural moduli is driven by the values in the bending matrix.

When there is a moment applied to the laminate, is no longer null.

Also, = whereas =

The important thing to notice is that in the formula of , distances from the laminate mid-plane (Zk) have an influence of to the power 3. As a consequence, the values for the different 0° plies don’t compensate each other like they were doing in the formula of where distances were taken into account without any power. This is why different flexural moduli can be noticed.

As an example, it is going to be considered two more simple laminates: a (0/90)2S and a (02/902)S. In the end, the result for (0/90)2S and (02/902)S laminates can be extrapolated to (0/90)4S and (04/904)S laminates. It is indeed just a matter of thickness but the mechanisms remain the same.

|  |  |
| --- | --- |
|  | 0° |
|  | 90° |
|  | 0° |
|  | 90° |
|  | 90° |
|  | 0° |
|  | 90° |
|  | 0° |

|  |  |
| --- | --- |
|  | 0° |
|  | 0° |
|  | 90° |
|  | 90° |
|  | 90° |
|  | 90° |
|  | 0° |
|  | 0° |

4h

4h

Figure: (0/90)2S laminate and a (02/902)S laminate.

Discussion about the (0/90)2S laminate:

Only the 0° plies have an influence in the value of the flexural moduli. As a consequence, if the further 0° ply from the laminate mid-plane is considered, the value of the bending moment “coefficient” Zk3 – Zk-13 would be: (4h)3 – (3h)3 = 37h3.

Now, if the second 0° ply is considered, the value of the bending moment “coefficient” Zk3 – Zk-13 would be: (2h)3 – (h)3 = 7h3.

Discussion about the (02/902)S laminate:

The first 0° ply is at the same position as in the (0/90)2S configuration. As a consequence the value of the bending moment “coefficient” Zk3 – Zk-13 remains the same: 37h3.

However, the second ply is not as close to the laminate mid-plane as in the previous configuration. Therefore, the value of the bending moment “coefficient” Zk3 – Zk-13 is different and would be:   
(3h)3 – (2h)3 = 19h3.

As a conclusion, it has been proven that because of the to the power 3 distances, the positions of the 0° plies in the laminate have an influence on the value of the flexural moduli.

In fact, the further the 0° plies are from the centre, the bigger the flexural moduli is (the laminate is harder to bend).

These results can be extrapolated to the (0/90)4S and (04/904)S laminates: they are going to have different flexural modulus and the second one is going to be harder to bend

SI LE MOMENT EST APPLIQUE SELON LES 0° plies.

And on the contrary, the values of Ex and νXY are driven by the values in the extensional stiffness matrix. As it can be seen in the previous part of the question, the calculation shows that the ply position h does not appear in the final result because as it has been said, each ply compensate each other (distances are taken into account without any power). As a consequence, the Poisson’s ratio and the Young’s modulus for the two laminates remain the same.

(c) Coupons machined from (0/90)4S and (04/904)S laminates are loaded parallel to the 0° plies to failure. Describe the sequence of damage development expected to be observed in the two coupons, and the effect of this damage on the Young’s modulus and Poisson’s ratio of the coupons. Draw attention to any differences you would expect between the coupons and explain why these differences occur.

Easy, cf lecture4, section 1.2.3.